International Journal of Theoretical Physics, Vol. 12, No. 5 (1975), pp. 299–320

Crossing Relations Derived from (Extended) Relativity

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Received: 3 *March* 1974

Abstract

Recently, Special Relativity has been straightforwardly extended to Superluminal inertial frames and faster-than-light objects. The 'Extended Relativity' theory not only allowed building up a self-consistent 'classical theory' of tachyons, but reveals itself useful also for the understanding of standard (subluminal) physics, i.e. of usual particles. In this paper, it is shown that Extended Relativity allows: (i) deriving the usual 'Crossing Relations' of elementary particle (high-energy) physics; and (ii) deriving the CPT-covariance theorem as a particular case of G-covariance (i.e., covariance under the new group of Generalised Lorentz transformations, both subluminal and Superluminal).

In this framework, the 'Analyticity' postulate is unnecessary: it is better substituted by the G-covariance requirement.

Moreover, new 'crossing-type' relations are predicted on the basis of mere Extended Relativity. They may well serve as a test for relativistic covariance of 'force fields' like strong interactions and, particularly, weak interactions, and possible new 'interaction fields' (which *a priori* are not relativistically covariant).

1. *Introduction*

1.1. *Extended Relativity*

Recently, Special Relativity (SR) has been straightforwardly extended (Parker, 1969; Mignani & Recami, 1973a, 1973b, 1973c; Recami, 1973; Recami & Mignani, 1974) to Superluminal inertial frames and to faster-than-light objects (tachyons).* Extended relativity immediately allowed building up a 'classical theory' of tachyons (Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973a, 1973b, 1973c, 1973d; Recami, 1973; Recami & Mignani, 1974).

However, here we want to deal with the consequences that Extended

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* *Note added in proofs*: Different theories have been proposed (Antippa & Everett, 1973; Goldoni, 1973) but these theories, however, *violate* the usual postulates of relativity.

Relativity Theory bears for usual (subluminal) matter, i.e., for usual particles (bradyons) and for photons.

Let us emphasise that standard SR contained as an additional, implicit postulate (*a priori* unjustified) the assumption that speeds greater than light speed c are not allowable. It is enough to get rid of that postulate (arbitrary from the theoretical viewpoint) in order to get plainly the Extended SR (Mignani & Recami, 1973a, 1973b, 1973c; Recami, 1973; Recami & Mignani, 1974.)

The logical framework of Extended SR may be found, e.g., in Mignani & Recami (1973a) and Recami & Mignani (1973, 1974).

The generalised Lorentz transformations GLT (both subluminal LT, and Superluminal SLT) may be found, e.g., in Mignani & Recami (1973a, 1973b, 1973c), Recami (1973) and Recami & Mignani (1974). Here, let us report only what follows:

(i) the new (Mignani & Recami, 1973b; Recami, 1973) group G of GLT's is:

$$G \equiv \mathscr{L}_1 \cup \mathscr{L}_2 \cup \mathscr{L}_3 \cup \mathscr{L}_4 \tag{1.1.1}$$

where‡

$$\left.\begin{aligned} \mathscr{L}_1 &\equiv \{+\Lambda_<\}; & \mathscr{L}_2 &\equiv \{-\Lambda_<\} \\ \mathscr{L}_3 &\equiv \{+i\Lambda_>\}; & \mathscr{L}_4 &\equiv \{-i\Lambda_>\} \end{aligned}\right\} \tag{1.1.2}$$

In equations (1.1.2), matrices $\Lambda_< \equiv \Lambda_<(\beta^2 < 1)$ are the usual, orthochronous (homogeneous†) LT's, and $\Lambda_> \equiv \Lambda_>(\beta^2 > 1)$ are (complex) matrices *formally* identical to the $\Lambda_<$'s, but corresponding to values of β such that $|\beta| > 1$.

(ii) The SLT's result (Mignani & Recami, 1973b; Recami, 1973) to have the form: SLT = $\pm i\Lambda_>$. It is easy to verify, e.g., that the product of two SLT's yields a subluminal LT.

(iii) While subluminal LT's do *not* change (as usual) the four-vector type, on the contrary the SLT's transform time-like vectors into space-like vectors, and vice versa (even if they too preserve the four-vector magnitudes–except for the sign).

(iv) The extended velocity composition law (Baldo *et al.*, 1970; Olkhovsky & Recami, 1971; Recami & Mignani, 1972; Mignani & Recami, 1973a, 1973b, 1973c, 1973d; Recami, 1973) bears for instance the consequences listed in Tables 1 and 2, which will be useful to us in the following.

1.2. *Usefulness of Extended Special Relativity*

The most important contributions coming from Extended SR (ESR), to the understanding of the usual physical domain, seem to be the following:

(I) A clarification of the connection Matter ↔ Antimatter. For this point, we refer to Mignani & Recami (1973a, 1973b, 1973c) and Recami (1973).

† We are not considering space-time translations.

‡ *Note added in proofs*: For a discussion of SLT's see also Mignani & Recami: *Letters Nuovo Cimento* 9, 357 (1974).

TABLE 1. Consequences of the extended velocity composition law for velocity magnitudes

$u^2 < c^2$	$v^2 < c^2$	$\Rightarrow v'^2 < c^2$
	$v^2 = c^2$	$\Rightarrow v'^2 = c^2$
	$v^2 > c^2$	$\Rightarrow v'^2 > c^2$
$u^2 = c^2$	$v^2 \Rightarrow c^2$	$\Rightarrow v'^2 = c^2$
$u^2 > c^2$	$v^2 < c^2$	$\Rightarrow v'^2 > c^2$
	$v^2 = c^2$	$\Rightarrow v'^2 = c^2$
	$v^2 > c^2$	$\Rightarrow v'^2 < c^2$

(II) A derivation of the CPT-covariance as a particular case of G-covariance (covariance under the Group G). We shall briefly show this point in Section 2. See also Mignani & Recami (1973a, 1973b, 1973c) and Recami (1973).

(III) A derivation of the usual '*crossing relations*', of elementary particles (high-energy) physics, from the mere (Extended) Special Relativity. Such a derivation (see Section 3) is the main aim of this paper. Moreover, existence of new, *similar* 'relations' will be predicted.

Before going on, let us remember (Recami & Mignani, 1974) that for free particles, in the *four-momentum space*, we have (Mignani & Recami, 1973a, 1973b, 1973c; Recami, 1973):

$$
\begin{aligned}
&E^2 - \mathbf{p}^2 \equiv p^2 = m_0^2 > 0 && \text{for bradyons } (B) \\
&E^2 - \mathbf{p}^2 \equiv p^2 = m_0^2 = 0 && \text{for luxons } (l) \\
&E^2 - \mathbf{p}^2 \equiv p^2 = -m_0^2 < 0 && \text{for tachyons } (T)
\end{aligned}
\qquad (1.2.1)
$$

where m_0 is always *real* (Baldo *et al.*, 1970; Recami & Magnani, 1972, 1974; Mignani & Recami, 1973a, 1973d). Generical hyperboloids corresponding to the previous three cases (*time-like, light-like, space-like,* respectively) are represented in Fig. 1.

Let us now *define* (Dhar & Sudarshan, 1968; Olkhovsky & Recami, 1969; Recami, 1969a, 1970, 1973; Mignani & Recami, 1973a, 1973b, 1973c) the negative energy points of hyperboloids in Figs. 1(a), (b), (c) as representing the possible kinematical states of the ANTIPARTICLE (of the *particle* represented by the corresponding positive-energy points). We shall see the reason of such a definition; in particular, it will be shown to coincide with the usual definition in the bradyonic case. By the way, let us remember that (*in the usual language*) the operation of 'changing *particles into antiparticles*' and vice versa, is the '$\hat{C}\hat{P}\hat{T}$' operation (see the following).

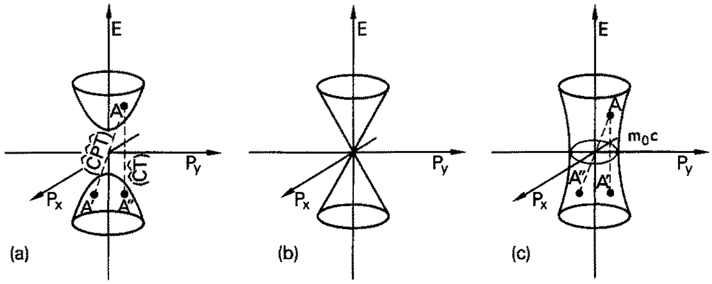

Figure 1–Representation of the hyper-surfaces $E^2 - \mathbf{p}^2 = p^2$, for: (a) BRADYONS, with $p^2 \equiv m_0{}^2 > 0$ (*time-like* case); (b) LUXONS, with $p^2 \equiv m_0{}^2 = 0$ (*light-like* case); (c) TACHYONS, with $p^2 \equiv - m_0{}^2 < 0$ (space-like case), where m_0 is *always real*. In Fig. (a), the points A' and A'' represent the particle kinematical states obtained by applying the operations $\hat{C}\hat{P}\hat{T}$ and $\hat{C}\hat{T}$, respectively, to the kinematical state A. *In the case when we confine ourselves* to subluminal frames and to usual LT's, then it happens that the 'matter' or 'antimatter' character is invariant for B's, but relative to the observer for T's. When eliminating previous restriction, we may pass from particles to their antiparticles (through GLT's) even in the case of bradyons.

1.3. *Digression–The 'Reinterpretation Principle' (RIP) for Tachyons*

The kinematical state of a generic *free* particle (with parameter m_0) will be represented by a point on one of the hyper-surfaces in Figs. 1. And the kinematical states of that free particle with respect to all *the subluminal inertial frames* will be represented by all the points of *the same* surface sheet. In fact, usual (subluminal) LT's do not effect transitions from a sheet to another one.

A simple look at Fig. 1(c), which shows a *connected* hypersurface, imposes the following *digression.*

For tachyons, (subluminal) LT's will exist that operate with continuity transitions from upper-semispace points to lower-semispace points. In other words, it may seem that a tachyon, regularly appearing to observer O as having positive energy (see point A of the upper semi-hyperboloid), will appear to other observers O' as bearing negative energy (see, e.g., point A' of the lower hyperboloid).

However, if a LT (see e.g., Baldo *et al.* (1970), Recami & Mignani (1972), Mignani & Recami (1973a, 1973b, 1973c, 1973d) and Recami (1973)) is such to invert the energy sign, the same LT will in general invert the sign of any other tetravector's fourth component, associated with the same observed object; in particular, that LT will also invert the sign of time (Bilaniuk, Deshpande & Sudarshan, 1962; Recami, 1969a). This fact is visually shown, e.g., in Fig. 15 of Mignani & Recami (1973a). Namely, if a tachyon moves, e.g., along the x-axis with positive velocity U with respect to us, the above-mentioned sign inversions happen (Sudarshan, 1969a; Baldo, Fonte & Recami, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973b, 1973c, 1973d; Recami, 1973) for all boosts corresponding to positive velocities $u > c^2/U$ (along the x-axis, and with reference to us).

In conclusion, if a tachyon is expected to show *negative* energy relative to

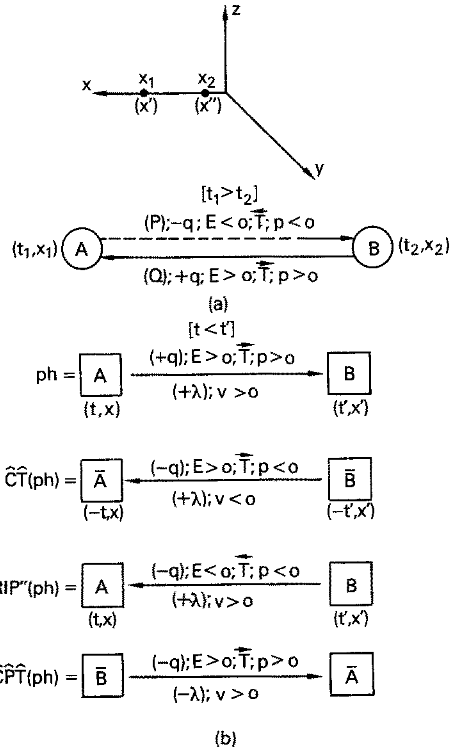

Figure 2—Figure (a) represents the exchange from A to B of a particle P with *negative* energy (and 'charge') and travelling backwards in time ($t_2 < t_1$). Such a process will appear nothing but the exchange *from B to A* of a (standard) particle Q with *positive* energy (and 'charge'), travelling forward in time. Particle Q may be shown to be (maybe except for the helicity) the *antiparticle* of the initial particle: $Q = \bar{P}$. See the text. Figure (b) shows a certain phenomenon *ph*, i.e., the exchange from emitter A to absorber B of a certain particle, and the transformations on *ph* (and on A, B) operated *respectively: by* $\hat{C}\hat{T}$, *by* the 'reinterpretation procedure' (RIP: see the text) used in case (a), *and by* $\hat{C}\hat{P}\hat{T}$.

a certain observer, it is also expected to appear to the same observer as moving *backwards in time* (with respect to the time-arrow univocally determined by usual macrosystems behaviour). It is very easy to convince ourselves that those two paradoxical occurrences allow a quite orthodox reinterpretation, when they are (as they actually are) contemporary. That '*Reinterpretation Principle*', that we shall call 'RIP', has been first forwarded by Bilaniuk *et al.* (1962) and Arons & Sudarshan (1968), in the spirit of previous interpretations by Dirac (1930), Stückelberg (1941) and Feynman (1949).

Namely, let us suppose (see Fig. 2(a)) that a particle P, with negative energy (and, e.g., 'charge' $-e$) and travelling backwards in time, is emitted by A at time t_1 and absorbed by B at time $t_2 < t_1$. Therefore, at time t_1, object A 'loses' negative energy and 'charge' $-e$. i.e. *gains* energy and 'charge' $+e$; and, at time $t_2 < t_1$, object B 'gains' negative energy and 'charge' $-e$, i.e. *loses* energy and 'charge' $+e$. Such a physical phenomenon will of course appear nothing but the exchange *from B to A* of a (standard) particle Q, with positive energy (and 'charge' $+e$) and travelling forward in time.

We have therefore seen that Q has opposite 'charge' to P; this means that 'RIP' operates, among the others, a charge conjugation $\hat{C}$. A closer inspection of 'RIP' (see Fig. 2(b), and cf. Recami & Mignani (1974)) tells us that effectively

$$\boxed{\text{'RIP'} \equiv \hat{C}\hat{E}\hat{p}} \tag{1.3.1}$$

where by $\hat{E}$ and $\hat{p}$ we mean the operations of energy-reversal and momentum-reversal, respectively (Arons & Sudarshan, 1968; Dhar & Sudarshan, 1968; Glück, 1969; Olkhovsky & Recami, 1969; Recami, 1969a, 1970; Sudarshan, 1969a; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). Notice that, in our terminology, $\hat{C}$ means conjugation of *all* charges (e.g., also of magnetic charge, if it exists).

In the present case of tachyons, we call Q the anti-particle of P:

$$Q = \bar{P}, \qquad (v^2 > c^2)$$

thus proving (in the tachyonic case) what was defined in Section 1.2.

In other words: let us consider a tachyon T travelling, e.g., along the x-axis, and a continuous series of subluminal frames, s, moving collinearly with our frame s_0. Let us call s_∞ the (critical) frame in which T becomes *transcendent* (i.e., with infinite velocity). As we go from s_0 to s_∞, tachyon T appears with increasing velocities. As we by-pass s_∞, with a LT that we shall call $\bar{L}$, the new frames *should* observe a tachyon T (still with positive 'charge') moving backwards in time and carrying negative energy. On the basis of the Dirac-Stückelberg-Feynman-Sudarshan 'Reinterpretation Principle' (Bilaniuk *et al.*, 1962)† however, we shall change the previous statement into the following one: 'As we by-pass the critical frame s_∞, the new frames will judge the observed particle as the *antitachyon*,‡ $\bar{T}$ (now with negative 'charge'), travelling in the opposite direction'.

In fact, let us, for example, consider the (subluminal) LT = $\tilde{L}$ making transition from A to A' of Fig. 1(c) (by the way, $\tilde{L}$ is the x-axis boost with positive relative velocity $u \equiv u_x = 2V_x/(1 + V_x^2/c^2)$, if V is the velocity of the considered tachyon in the kinematical state A). It is easy to see (cf. Table 2) that $\tilde{L}$ acts kinematically *on the observed tachyonic object* T as the product $\hat{E}\hat{T}\hat{v}$, where $\hat{v}$ is the velocity-reversal operation. When we apply 'RIP', we eventually find that the second frame observes—as regards the tachyon analysed—the same effect as produced (in the first frame) by a mere $\hat{C}\hat{T}$ operation (Sudarshan, 1969b),

$$(\hat{C}\hat{E}\hat{p})(\hat{E}\hat{T}\hat{v}) \rightarrow \hat{C}\hat{T}$$

† Such a Principle is a *necessary* 'Third Postulate' even in *standard* Special Relativity: cf. Mignani & Recami (1973a) and Recami & Mignani (1974).

‡ Cf. also Arons & Sudarshan (1968).

applied only to the observed object T. Such a $\hat{C}\hat{T}$-action, as shown by Fig. 2(b), does *prove* our previous statement between quotation marks. The emerging fact that, given a particle P, the concept of 'antiparticle', $\bar{P}$, is a purely relativistic one will be soon revisited.

Previous analysis prompted us to introduce into Special Relativity theory a *Third Postulate* (besides the two standard ones: (i) Relativity Principle (PR), and (ii) isotropy of space and omogeneity of space-time): i.e., the *'Reinterpretation Principle'* (RIP), in the form (Sudarshan, 1969b, 1970; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973b, 1973c, 1973d; Recami, 1973): 'Physical signals are actually transported only by positive energy objects (i.e., by the objects that appear to us as carrying positive energy and going forward in time)'. The meaning of such a Principle within Information Theory is straightforward. The 'RIP' inserts *harmoniously* into Special Relativity: it was indeed shown to be *necessary* to the self-consistency of standard Special Relativity and of Generalised Special Relativity (Mignani & Recami, 1973a). For example, the above-mentioned Generalised velocity composition law (see Baldo *et al.* (1970), Recami & Mignani (1972), Mignani & Recami (1973a, 1973b, 1973c, 1973d) and Recami (1973)) holds for the *reinterpreted* objects (Sudarshan, 1972; Recami & Mignani, 1974). The same happens, e.g., for the electric charge q; that is to say, a LT making transition between a frame f_1, 'preceding' the critical frame s_∞, and a frame f_2, 'following' s_∞ (i.e., such to 'overcome' the critical velocity), will automatically yield the final electric charge shown by the *reinterpreted* objects (Recami & Mignani, 1974). In fact, a LT, being such to invert the fourth-components' sign, will also invert the sign of charge density, ρ, and therefore the particle (total) charge q:

$$\rho \to -\rho; \qquad q \to -q \tag{1.3.2}$$

where we *defined:*

$$q \equiv + \int \rho |dV| \tag{1.3.2'}$$

This agrees with the fact that the above-considered LT transforms tachyons into antitachyons, and vice versa. We shall come back to the RIP also in Section 2.3.

2. *The 'RIP' for Bradyons, and the CPT-Theorem*

2.1. *Antimatter and Matter*

The RIP is appropriate in the bradyonic case as well (Recami, 1969a; Recami & Mignani 1974). Namely, a particle P in the kinematical state corresponding to a point of the lower hyperboloid (Fig. 1a), will be shown to appear as the antiparticle $\bar{P}$ of P *in the usual sense.* It is quite interesting that, once the notion of *particle* is introduced (as usually used in Special Relativity), merely from Special Relativity itself the concept of *anti-particle* follows (Recami, 1969a, 1973; Mignani & Recami, 1973b, 1973c). Since 1905, on the basis of the double sign entering relation

$$E = \pm\sqrt{(\mathbf{p}^2 + m_0{}^2)} \tag{2.1.1}$$

the existence, for any particle, of its antiparticle could have been expected, provided the RIP had been used.

Moreover, let us emphasise that—*when we limit ourselves to subluminal frames*—the clean separation between *matter and antimatter* is confined only to bradyons, owing to the fact that the hyperboloid in Fig. 1(a) consists† of two disconnected sheets (Dhar & Sudarshan, 1968; Olkhovsky & Racami, 1969; Recami, 1969a, 1970; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). On the contrary, in the case of tachyons, the character matter/antimatter is no more absolute, but *relative* to the (subluminal) observer (Dhar & Sudarshan, 1968; Olkhovsky & Recami, 1969; Recami, 1969a, 1970; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d).

However, if we *consider also Superluminal frames*, since the product of two suitable SLT's may yield (Mignani & Recami, 1973a, 1973b, 1973c; Recami, 1973) a GLT of the type $-\Lambda_< \equiv (\hat{P}\hat{T})\Lambda_<$, we may get, by means of a GLT, the transition matter $\rightleftarrows$ antimatter (see the definition in Section 1.2) even *for bradyons.* Let us consider the *particular, non-orthochronous* LT that is usually called $\hat{P}\hat{T}$:

$$-\tilde{\Lambda}_< \equiv \hat{P}\hat{T}; \qquad [\tilde{\Lambda}_< \equiv \mathbf{1}]$$

The 'strong reflection', or 'total inversion', is of course an element of G. Here we meet the important point that follows: *in a universe with 'charges',* the Generalised Lorentz transformation that we called $\hat{P}\hat{T}$ does effectively produce the exchange particle $\rightleftarrows$ antiparticle, and must be *actually* considered a $\hat{C}\hat{P}\hat{T}$ operation (Sakurai, 1964a; Berestetsky *et al.*, 1971; Sudarshan, 1968.)

$$\boxed{-\tilde{\Lambda}_< \equiv \mathbf{1}; \qquad -\mathbf{1} \equiv \hat{C}\hat{P}\hat{T}} \tag{2.1.2}$$

where, as before, $\hat{C}$ means 'inversion of all (additive) "charges" '.

Therefore (in a universe with 'charges'), the element of G:

$$-\mathbf{1} \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

when correctly thought as acting on every relativistic four-vector of interest (and not only on four-position), results in the $\hat{C}\hat{P}\hat{T}$ operation. As a conclusion, *the* $\hat{C}\hat{P}\hat{T}$ *operation* (rather than PT) *has to be considered an element of* G*:*

$$-\mathbf{1} \overset{(\mathrm{RIP})}{\equiv} \hat{C}\hat{P}\hat{T} \tag{2.1.3}$$

† At the finite, at least.

We shall deepen such considerations in the following.

Here, let us remember that the transition from a particle P to its antiparticle $\bar{P}$ (in the same kinematical state, *as seen by the same observer*, and *with emitter and absorber interchanged*), is performed by the $\hat{C}\hat{P}\hat{T}$ operation: see Fig. 2(b). Strictly speaking, in Fig. 1(a) the *true antiparticle* state of A is A', and in Fig. 1(c) is A''. Since CPT is known to *change* the helicity-sign, the *'true antiparticle state'* of the particle with helicity $+\lambda$ has helicity $-\lambda$.

Such an operation may reduce to $\hat{C}\hat{P}$ or to $\hat{C}$ when the interactions that P and $\bar{P}$ undergo are $\hat{T}$ or $\hat{P}\hat{T}$ covariant, respectively. An example of the last case is the one of (purely) electromagnetic interactions.

2.2. $\hat{C}\hat{P}\hat{T}$ *Covariance and the 'RIP'*

We have just seen that, since $-\mathbf{1}$ is a chronotopical 'rotation', i.e., a GLT, relativistic physical laws are expected to be covariant under the $\hat{C}\hat{P}\hat{T}$ symmetry. Since the 'total inversion' is a subluminal LT (corresponding to $\beta = 0$ but $\phi = \pi$), the previous statement should refer also to usual relativistic laws, even if not yet written in G-covariant form. This agrees with what is found on a *formal basis*, e.g., in Sakurai (1964a), Sudarshan (1968) and Berestetsky *et al.* (1971).

It is important to notice that, as we saw in the last sub-section, in the case of bradyons the transition from an 'upper' point A to a 'lower' point A' of Fig. 1(a) is actually performed by a non-orthochronous LT of the type $-\Lambda_<$, e.g., by $-\mathbf{1}$ itself.

But, as we mentioned before, the GLT $= -\mathbf{1}$ that we initially called $\hat{P}\hat{T}$ does change the sign not only of t and $\mathbf{x}$, but also of all other four-vector components, and in particular of energy E and momentum $\mathbf{p}$ of any considered object:

$$\hat{P}\hat{T} \to -\mathbf{1} \equiv \hat{P}\hat{p}\hat{T}\hat{E} \qquad (2.1.3\ bis)$$

Consequently, if we remember equation (1.3.1), i.e., that 'RIP' $= \hat{C}\hat{E}\hat{p}$, we may draw the conclusion that strong reflection and RIP yield (Arons & Sudarshan, 1968; Dhar & Sudarshan, 1968; Baldo & Recami, 1969; Glück, 1969; Sudarshan, 1969a, 1969b; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d)

$$\hat{C}\hat{E}\hat{p}(\hat{P}\hat{p}\hat{T}\hat{E}) = \hat{C}\hat{P}\hat{T} \qquad (2.2.1)$$

Since equation (2.2.1) obviously holds not only for B's but also for T's, we have thus proved equations (2.1.2) and (2.1.3). See also Figs. 2. Notice that, in our theory, $\hat{C}\hat{P}\hat{T}$ is a *linear operator* (Sudarshan, 1969b), as well as *all* GLT's.

In such considerations (cf. also Section 1.3), when *assuming* RIP, it is *always* possible to write the relation:

$$\boxed{\hat{C} \overset{(\mathrm{RIP})}{\equiv} \hat{v}\hat{E} \equiv \hat{p}\hat{E}} \qquad (2.2.2)$$

which is very interesting for the *physical* understanding of 'charge conjugation'.

By the way, notice that also in the case of bradyons—by means *of*: (i) suitable GLT's of the type $-\Lambda_<$; (ii) their kinematical effects (see Table 2) on an observed object; and (iii) the RIP—one may eventually get that the *second frame* observes (*as regards the analysed bradyon*) the same effects as produced (*in the first frame*) by a $\hat{C}\hat{T}$ operation (Sudarshan, 1969b) *applied to the observed bradyon.* As in the case of tachyons, moreover, the generalised velocity composition law does hold for the objects *reinterpreted* by RIP. Analogously for the 'electric charge' density, transformed by the GLT considered.

It can be concluded that—since $-1 \equiv \hat{C}\hat{P}\hat{T}$ both for bradyons and tachyons —under the GLT 'strong reflection' particles (B or T) in the initial state of an interaction process will be transformed into antiparticles ($\bar{B}$ or $\bar{T}$) in the final state of the same interaction processes, and vice versa (Dhar & Sudarshan, 1968; Baldo & Recami, 1969; Glück, 1969; Sudarshan, 1969a, 1969b; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). For instance, the two reactions.

$$\left.\begin{array}{l} a + b \to c + d \\ \bar{c} + \bar{d} \to \bar{a} + \bar{b} \end{array}\right\} \qquad (2.2.3)$$

are the two different descriptions of *the same phenomenon* as seen by the two different inertial frames s_0 and $-s_0 \equiv (\hat{C}\hat{P}\hat{T})s_0$, respectively.

From the foregoing it follows that usual symbols like $\hat{P}$ and $\hat{T}$ have too *restrictive* a meaning (Sudarshan 1969b); one ought on the contrary to introduce symbols meaning the sign inversion produced by a GLT in *all* the tetravectors' fourth component (e.g. the *symbol*, $\hat{\hat{T}}$) and first three components (e.g.: $\hat{\hat{P}}$), and so on. We already specified that $\hat{C} \equiv \hat{\hat{C}}$ means the sign inversion of all additive 'charges.' It holds of course that:

$$\left.\begin{array}{ll} \hat{\hat{T}} \equiv \hat{T}\hat{E} \ldots; & \hat{\hat{P}} \equiv \hat{P}\hat{p} \ldots \\ \boxed{\hat{\hat{P}}\hat{\hat{T}} \overset{(\mathrm{RIP})}{=} \hat{C}\hat{P}\hat{T}} & \end{array}\right\} \qquad (2.1.3\ ter)$$

For completeness, let us *verify* explicitly the conclusions above also in the case of *tachyons.* First of all, from Table 2 it can be deduced that, by means of suitable transformations of the type $-\Lambda_<$, of their kinematical effects and of RIP, we may finally find that the second frame observes the same effects as produced (in the first frame) by the $\hat{C}\hat{P}\hat{T}$ operation. In the particular case when $-\Lambda_<$ is precisely the total inversion, then we obviously get the *true* $\hat{C}\hat{P}\hat{T}$, as stated above, i.e., transition from a tachyonic phenomenon to the $\hat{C}\hat{P}\hat{T}$-ed one (*both seen now in the same frame!*).

In other words,† if we observe from our frame s_0 a succession of Superluminal

† For greater clearness, let us remake the explicit illustration of a concrete example. Consider a tachyon T and a succession of subluminal frames (all moving collinearly with T, for simplicity). Let s_∞ be, as before, the frame observing T with divergent velocity. If a frame moving slower than s_∞ sees T travelling in a certain direction, then any frame moving faster than s_∞ will actually observe T as an antitachyon (e.g., with the opposite electric charge) travelling in the reversed direction (Sudarshan, 1969a, 1969b, 1970; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). Therefore by-passing frame s_∞ (in the above sense) implies the charge conjugation.

TABLE 2. Effect of GLT's on the sign of various four-vector components of an observed object, in the case of collinear motion along the x-axis. Both subluminal ($U \equiv U_x$) and Superluminal ($U \equiv U_x$) relative velocities are considered. Analogously, both bradyons (having velocities **v**, relative to the *first*, unprimed frame) and tachyons (having velocities **v**, relative to the unprimed frame) are also considered. For simplicity, *only* the cases $+\Lambda_<(\beta^2 < c^2)$ and $-i\Lambda_>(\beta^2 > c^2)$ are forwarded, as well as the *only* cases $v > 0$ and $V > 0$. Notice explicitly that only x-components of **v** or **V** are effective in this context.

Relative velocity	$\lvert u_x\rvert \equiv \lvert u\rvert < c;\ V_x > c$		$\lvert U_x\rvert \equiv \lvert U\rvert > c$				$\lvert u_x\rvert \equiv \lvert u\rvert < c;\ 0 < v_x < c$	
			$0 < v_x < c$		$V_x > C$			
	$u > 0$	$u < 0$	$U > 0$	$U < 0$	$U > 0$	$U < 0$	$u > 0$	$u < 0$
$\frac{\text{sign } x'}{\text{sign } x}$	+	+	+	−	± for $U \gtrless V_x$	−	± for $u \lessgtr v_x$	+
$\frac{\text{sign } t'}{\text{sign } t}$	± for $\frac{c^2}{V_x} \gtrless u$	+	± for $U \gtrless \frac{c^2}{v_x}$	−	+	−	+	+
$\frac{\text{sign } p_x'}{\text{sign } p_x}$	+	+	+	−	± for $U \gtrless V_x$	−	± for $u \lessgtr v_x$	+
$\frac{\text{sign } E'}{\text{sign } E}$	± for $\frac{c^2}{V_x} \gtrless u$	+	± for $U \gtrless \frac{c^2}{v_x}$	−	+	−	+	+
$\frac{\text{sign } v_x'}{\text{sign } v_x}$	± for $\frac{c^2}{V_x} \gtrless u$	+	± for $U \gtrless \frac{c^2}{v_x}$	−	± for $U \gtrless V_x$	−	± for $u \lessgtr v_x$	+
$\frac{\text{sign } j_x'}{\text{sign } j_x}$	+	+	+	−	± for $U \gtrless V_x$	−	± for $u \lessgtr \text{-}v_x$	+
$\frac{\text{sign } \rho'}{\text{sign } \rho}$	± for $\frac{c^2}{V_x} \gtrless u$	+	± for $U \gtrless \frac{c^2}{v_x}$	−	+	−	+	+

frames (*or vice versa*), for simplicity all moving with positive speeds $U \equiv U_x > c$, when we bypass the 'transcendent frame' S_∞—in other words, for *transfinite transformations* (cf., e.g., Fig. 2(b) in Recami & Mignani (1974)—we get the $\hat{C}$-symmetry. Thus, when we operate a 'rotation' (in the four-dimensional space-time†) aimed to reach the totally inverted frame $(\hat{P}\hat{T})s_0$, actually we reach the frame $(\hat{C}\hat{P}\hat{T})s_0$.

At this point, let us forward in Table 2 the effects of GLT's [both subluminal and Superluminal: $+\Lambda_<(\beta^2 < c^2)$ and $-i\Lambda_>(\beta^2 > c^2)$] on the sign of various physical quantities.

At last, let us investigate the case of *luxons.* The GLT's mapping the upper light-cone into the lower light-cone, and vice versa, are all the (subluminal, non-orthochronous) LT's of the type $-\Lambda_<(\beta \gtrless 0)$ and all the SLT's of the type $+i\Lambda_>(\beta > c)$ or $-i\Lambda_>(\beta < -c)$. Let us consider in particular the total inversion $\hat{C}\hat{P}\hat{T} \equiv -\Lambda_<(\beta = 0)$; we shall assume in this case too that it transforms a particle state into its antiparticle state. If the luxon bears a 'charge' (as neutrinos, that bear 'leptonic charge'), since $\hat{C}\hat{P}\hat{T}$ changes the helicity sign, a luxon (*neutrino*) with helicity $\lambda = -1$ will be transformed into its 'antiluxon' (Recami, 1969a) (*antineutrino*) with helicity $\lambda = +1$. Cf. Fig. 1(b).

In the case when luxons do not seem to bear any 'charge' (as photons), the distinction between 'photons' and 'antiphotons' (Recami, 1969a) becomes moot. We may say that, *if* a *photon* has helicity $\lambda = +1$, then *its antiphoton* will have $\lambda = -1$ (with source and detector interchanged).

2.3. *Conclusion*

Let us make some further considerations. Briefly, the cases of B's and T's in Section 2.2 tell us that: '*The right way for doing $\hat{P}\hat{T}$ is doing $\hat{C}\hat{P}\hat{T}$*' (Csonka, 1969; Sudarshan, 1969b; Mignani & Recami, 1973b; Recami, 1973). The $\hat{C}\hat{P}\hat{T}$-covariance, as already mentioned, is required by our *mere* 'extended PR' (when we don't confine arbitrarily ourselves to subluminal relative velocities) (Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d).

In the cases when $\hat{T}$-covariance is supposed to hold, we get as a corollary that: '*The right way for doing $\hat{P}$ is doing $\hat{C}\hat{P}$*', which expresses the essential teaching of Lee and Yang (1957). In fact, in the case considered, Relativity says that we can 'safely' (i.e., covariantly) reflect space only if we contemporarily apply $\hat{C}$, so to have particles changed into antiparticles.

Close inspection of cases dealt with in Section 2.2 reveals that their meaning vanishes (e.g., because of the intervening exchange of emission and absorption processes) if we cannot refer our B's or T's to some *interaction* (space-time) *regions*. For example, when a tachyon overcomes the divergent velocity, it passes from being, e.g., a tachyon T *entering* (a certain interaction region) to being an antitachyon $\bar{T}$ *outgoing* (from that interaction region) (Arons & Sudarshan, 1968; Glück, 1969; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). In conclusion, the *Third Postulate* (RIP) will be completed by proving that (Dhar & Sudarshan, 1968): '*Under a "transcritical" ' GLT*, when, e.g., the rôles of emissions and absorptions happen to be

† Compare, e.g., Figs. 2 in Recami & Mignani (1974).

interchanged, *any negative energy object in the initial (final) "state" corresponds physically to its positive-energy antiobject in the final (initial) "state"*, and vice versa', in the above sense.

Of course the *third postulate*—in order to be used for reinterpreting the GLT's effect—requires considering processes with both initial and final 'states' (Dhar & Sudarshan, 1968; Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973d). Therefore, we can see that Extended SR strongly suggests (Recami & Mignani, 1974) to deal with *interactions*, and *not* with *objects* (in quantum mechanical terminology, to deal with 'amplitudes' rather than with 'states' (Dhar & Sudarshan, 1968)).

Let us explicitly mention that it is actually the 'RIP' that allows us (Sudarshan, 1969b) to apply *directly* the non-orthochronous LT's to four-momentum vectors (as well as to the other four-vectors).

3. *Description of Nature, Physical Laws and GLT's*

In order to approach the problem of 'Crossing Relations' derivation, let us first premise what follows.

3.1. *Interactions and Objects*

We have already seen that, e.g., through suitable Generalised boosts, a particle moving along the positive x-axis (as seen in the first frame) may transform into its antiparticle (as seen in the second frame) travelling along the negative x-axis. Therefore, we argued (Section 2.3) that language about nature should refer to global *interaction processes* rather than to 'objects'. In fact, from such a viewpoint, the initial assertion has been given a precise meaning by writing that 'a particle in the *initial* (final) *state* may appear as its antiparticle in the *final* (initial) *state*, when observer is changed (by means of GLT's)'.

At this point, it is worthwhile to remark that Relativity does *not* require at all that two different observers forward the same *description* of the same phenomenon. It does require only that they find that phenomenon to be ruled by the same physical *laws* (generally speaking, *conservation laws*).

3.2. *Description and Laws*

In fact: Let us choose (Fabri, 1959; Agodi, 1973) a set $\mathscr{R}$ of certain, well-defined reference frames r, the set $\mathscr{P}$ of the phenomena p of Mechanics and Electromagnetism, and the set $\mathscr{D}$ of the descriptions d (of phenomena $p \in \mathscr{R}$ from frames $r \in \mathscr{R}$). All observers r are supposed to possess the same instruments, both physico-experimental and mathematico-theoretical (i.e., the same theory, too). Strictly speaking, one has to deal with the 'triads' dpr, elements of the set $\mathscr{D} \times \mathscr{P} \times \mathscr{R}$, cartesian product of the three sets considered. To the same p there correspond (Recami & Mignani, 1974) two descriptions d_1, d_2 in two frames r_1, r_2 and so on: given any *two elements* out of d, p, r, the correspondence between elements dpr must be assumed to be such that the third one is univocally fixed. We may write (Fabri, 1959; Agodi, 1973):

$$d_1pr_1 \xrightarrow{r_1 \to r_2} d_2pr_2; \qquad \text{i.e.:} \{r_1 \to r_2, p \to p \Rightarrow d_1 \to d_2\};$$
$$dp_1r_1 \xrightarrow{r_1 \to r_2} dp_2r_2; \qquad \text{i.e.:} \{r_1 \to r_2, d \to d \Rightarrow p_1 \to p_2\} \tag{3.2.1}$$

Let us define the subsets $\Delta_r \subset \mathscr{D}$:

$$d \in \Delta_r \Leftrightarrow dpr \in \mathscr{D}\mathscr{P}\mathscr{R} \equiv \mathscr{D} \times \mathscr{P} \times \mathscr{R} \tag{3.2.2}$$

Then, following Agodi (1973), we shall say that two frames r_1, r_2 are equivalent ($\doteq$) if Δ_r is *mapped onto itself* when passing from r_1 to r_2:

$$r_1 \doteq r_2 \Leftrightarrow \Delta_{r_1} \supseteq\!\!\!\subseteq \Delta_{r_2}$$
$$\Leftrightarrow \forall d \in \Delta_{r_1} \Rightarrow d \in \Delta_{r_2} \text{ and } \forall d' \in \Delta_{r_2} \Rightarrow d' \in \Delta_{r_1} \tag{3.2.3}$$

Such a condition operates an *exhaustive partition* of set $\mathscr{R}$ into subsets of equivalent frames. Conversely, given a frame r and a set $\mathscr{P}$ of phenomena, it is possible to build up the set $\mathscr{R}$ of frames equivalent to r. It is easy to verify that, given the set of Mechanical and Electromagnetic phenomena and an inertial frame, two classes of equivalent frames are respectively the set of standard (subluminal) inertial frames and the set $\mathscr{I}$ of *our* inertial (Baldo *et al.*, 1970; Recami & Mignani, 1972; Mignani & Recami, 1973a, 1973b, 1973c, 1973d; Recami, 1973) frames, both subluminal and Superluminal. Let us choose $\mathscr{R} = \mathscr{I}$. In our case, $\mathscr{R} = \mathscr{R}_s \cup \mathscr{R}_S$, where $\mathscr{R}_s \cap \mathscr{R}_S = \emptyset$, sets $\mathscr{R}_s$ and $\mathscr{R}_S$ being the class of subluminal inertial frames s and of Superluminal inertial frames S, respectively:

$$\mathscr{R}_s \equiv \{s\}; \qquad \mathscr{R}_S \equiv \{S\}$$

Now, moreover, $\mathscr{D} = \Delta_r \equiv \Delta, \forall_r \in \mathscr{R}$. It is immediately obvious that:

$$\forall p \in \mathscr{P} \Rightarrow \mathscr{D} = \mathscr{D}_s \cup \mathscr{D}_S, \qquad \text{with } \mathscr{D}_s \cap \mathscr{D}_S = \emptyset \tag{3.2.4}$$

where

$$d \in \mathscr{D}_{s,S} \Leftrightarrow dpr \in \mathscr{D}\mathscr{P}\mathscr{R}_{s,S} \Leftrightarrow r \in [\{s\}, \{S\}] \Leftrightarrow$$
$$\Leftrightarrow r = [s, S]$$

Notice that, in passing from a s to a S, we have that $\mathscr{D}$ is mapped—as required—onto itself, but in such a way that $\mathscr{D}_s$ goes onto $\mathscr{D}_S$, and vice versa.

Given a phenomenon p, if d_1 and d_2 are its descriptions in the frames r_1, r_2, respectively, and if transformation L is such that

$$Lr_1 = r_2 \tag{3.2.5}$$

we shall consequently use the convention of writing:

$$Ld_1 = d_2 \tag{3.2.5 bis}$$

Let us suppose we have a *criterion*, C, for a given description d to belong to the set $\mathscr{D}$ of the descriptions of phenomena $p \in \mathscr{P}$ from the frame $r \in \mathscr{R}$; we write:

$$C(d) \text{ verified } \Leftrightarrow d \in \mathscr{D} \tag{3.2.6}$$

We shall call C a 'good criterion' if it holds for any d:

$$\forall d \in \mathscr{D} \Rightarrow C(d) \text{ verified } \Leftrightarrow d \in \mathscr{D} \tag{3.2.7}$$

It follows that C is *covariant* in form under any L:

$$C(Ld) \text{ verified } \Leftrightarrow Ld \in \mathscr{D} \tag{3.2.7$'$}$$

We shall by definition call C (or better the union of the possible, various 'good criteria' $C_1, C_2 \ldots$) the *ensemble of physical laws* of phenomena $p \in \mathscr{P}$ as seen by frames $r \in \mathscr{R}$. Conversely, a proposition will be considered a physical law if it is a part of C. *In other words*, given $\mathscr{R}$ and $\mathscr{P}$, we define 'physical law' as any proposition regarding a $p \in \mathscr{P}$ which is covariant within $\mathscr{R}$.

3.3. *An Example*

Going back to Section 3.1, let us for instance emphasise that electric charge —as well as energy, and so on—of an isolated system is *not* required to be a relativistic invariant; but only to be constant (as seen by any observer) during the system transformations.

It is instructive to analyse an explicit example.† Let us consider a positive charged particle, a, that, with respect to a first observer O_1, decays into a neutral particle, c, and another positive charged particle, b, having in general different velocity. It is possible to find another observer O_2, with respect to whom, e.g., the outgoing particle b behaves as an incoming antiparticle, $\bar{b}$, bearing a negative charge. Observer O_2 will judge the process as a 'resonance' formation.‡ Frame O_1 observes a total electric charge +1, whilst O_2 a total charge *zero*. Both observers, however, will agree about electric charge conservation law to be verified in the observed process. Moreover, before interaction O_1 will see *one* particle, while O_2 *two* particles. Therefore, the very number of particles (e.g., of tachyons), at a certain instant of time, is *not* Lorentz invariant (Feinberg, 1967; Baldo *et al.*, 1970). However, *the total number of particles* (e.g., of tachyons) *participating to the reaction* (in both initial *and* final states) *is Lorentz invariant,* due to the very features of RIP (Baldo *et al.*, 1970). Again, we are prompted to build the physical theory in terms of 'reaction processes' rather than of 'objects'. Such a suggestion is of great philosophical meaning.

Before closing this sub-section, let us underline that usual proofs of electric charge invariance hold *only* for subluminal, orthochronous LT's applied to bradyons (Pauli, 1921). We have already shown that the charge of a particle may change sign under GLT's.

† See also, e.g., Fig. 18 in Recami & Mignani (1974).
‡ Or as an 'annihilation process'.

4. *Crossing Relations, and all That*

4.1. *G-Invariant Amplitudes in Special Relativity*

Let us consider, e.g., a large number of two-body (*macroscopic*) scatterings

$$A + B \to \text{everything} \tag{4.1.1}$$

in a certain frame r_0.

Observer r_0 will measure the probability $\Delta W(\theta_C, \phi_C)$ of the process

$$A + B \to C + D \tag{4.1.2}$$

for C contained in a certain (small) solid angle around direction θ_C, ϕ_C by calculating the ratio between the number of the *good* events (4.1.2) and the total number N of events (4.1.1).

When going from r_0 to r', quantity ΔW remains, of course, invariant (for *conservation of events,* and the RIP behaviour); it will now give the probability of the transformed process (i.e., of (4.1.2) *as seen* by r), with the transformed C contained in the *transformed* solid angle:

$$\Delta W(\theta'_C, \phi'_C, \ldots) = \Delta W(\theta_C, \phi_C, \ldots) \tag{4.1.3}$$

In other words, if $d\sigma/d\Omega \equiv \sigma(\theta, \phi)$ is the ('good' processes) *differential, invariant* (Sakurai, 1964) *cross-section*, our (invariant) differential probability dW coincides with $d\sigma \equiv \sigma(\theta, \phi) \,.\, d\Omega$, except for an invariant flux normalisation. Of course, $d\sigma$ is also *invariant*: (Sakurai, 1964; Messiah, 1965)

$$\sigma(\theta, \phi) \,.\, d\Omega = \sigma'(\theta', \phi') \,.\, d\Omega'$$

or better:

$$d\sigma(\theta, \phi, \ldots) = d\sigma(\theta', \phi', \ldots) \tag{4.1.4}$$

Quantity $d\sigma$ may be written (Sakurai, 1964) as the product of an (invariant) kinematical factor times another factor, I, invariant as well, which specifies the interaction 'dynamics'.

Let us now pass to the *microscopic* physics. What was previously said is still valid, under the *only* assumption that the elementary interaction is relativistically *covariant. Strictly speaking, this is known to happen only for electromagnetic interactions.*† For relativistically covariant interactions, the above-mentioned factor I may be written (Sakurai, 1964) as the square modulus of an *invariant, complex* function A of all quantities characterising initial and final states:

$$I \equiv |A|^2 \tag{4.1.5}$$

Quantity A is known as the *invariant Amplitude* of the process when its *variables* are explicitly chosen so as to be at least invariant (Sakurai, 1964b) under subluminal LT's (in such a case, they will change sign under SLT's).

† Maxwell equations have been written in G-covariant form in Mignani & Recami (1973a, 1973d) and Recami & Mignani (1974).

When considering a two-body to two-body reaction, besides the (usual) conservation laws, one meets the new law (i.e., the *specific law* of that process)

$$dW = dW(\theta_C, \phi_C, \ldots) \tag{4.1.6}$$

as clarified in Section 3.2. By the way, equation (4.1.6) also expresses the *G-invariance* of the global number of particles participating in an interaction process (in initial *plus* final states). In the microphysics case, equation (4.1.6) is better substituted by the *law:*

$$A = A(s, t, \ldots) \tag{4.1.6$'$}$$

where $s, t, \ldots$ represent all the so-called 'invariant variables' on which A must depend. In particular, s and t are the 'Mandelstam variables', i.e. (Chew, 1962; Hagedorn, 1963; Roman, 1969):

$$\left.\begin{aligned} s &\equiv (p_A + p_B)^2 \equiv (p_C + p_D)^2 \\ t &\equiv (p_A - p_C)^2 \equiv (p_B - p_D)^2 \end{aligned}\right\} \tag{4.1.7}$$

These quantities (being four-vector magnitudes squared) are actually *invariant* under subluminal LT's ($\pm\Lambda_<$), and *change sign* under SLT's ($\pm i\Lambda_>$):

$$(\text{SLT})s = -s; \qquad (\text{SLT})t = -t \tag{4.1.8}$$

Sometimes (Chew, 1962; Hagedorn, 1963; Roman, 1969) it happens that $A(s, t, \ldots) \equiv A(t, s, \ldots)$.

4.2. *Effects of GLT's on Reaction Process Descriptions*

When the same interaction process p gives rise to different descriptions from different observers, i.e., appears as different scattering processes $d_1(p)$ and $d_2(p)$ in different frames r_1 and r_2, then the (extended) principle of Relativity (PR) requires that $d_1(p)$ and $d_2(p)$ are ruled by the same dynamical law (4.1.6$'$). Therefore, because of relativistic covariance, *processes d_1 and d_2 present the same scattering amplitude $A = A(s, t, \ldots)$*, provided that *the physical meanings* (and possibly the signs) of the invariant variables $s, t, \ldots$ are accordingly *changed* (Chew, 1962; Hagedorn, 1963; Roman, 1969). Remember that GLT's and RIP do automatically save the validity of the usual conservation laws as well.

Now, let us consider subluminal and Superluminal boosts along the x-direction. We shall first consider only T's having $|V_x| > c$. It is then easy to observe (e.g., from Table 2) that:

(1) A subluminal boost, $L = \pm\Lambda_<$, applied to an interaction among T's (and/or luxons), allows transition (Baldo *et al.*, 1970) from a certain process p either to p itself or to: (i) any scattering p' obtained by $\hat{C}\hat{T}$-ing (Sudarshan, 1969b) one or more particles; (ii) any scattering p'' obtained

by $\hat{P}$-ing no, one or more particles *and* $\hat{C}\hat{P}\hat{T}$-ing all the remaining particles; *provided that* processes p' and p'' are kinematically allowed (or, better, satisfy the conservation laws of energy, momentum, angular momentum and of all 'charges'). Scatterings p'' are nothing but the $\hat{C}\hat{P}\hat{T}$-ed ones of scatterings p'.

Besides:

(2) A Superluminal boost, $L = \pm i\Lambda_>$, applied to an interaction among B's (and/or luxons), allows transition from a certain process p either to p itself or to: (i) any scattering p' obtained by $\hat{C}\hat{T}$-ing (Sudarshan, 1969b) one or more B's with $v_x > 0$ *and* $\hat{C}\hat{P}\hat{T}$-ing all B's with $v_x \leq 0$; or vice versa; (ii) any scattering p'' obtained by either $\hat{P}$-ing or $\hat{C}\hat{P}\hat{T}$-ing all B's with $v_x > 0$ (and leaving unaffected all B's with $v_x \leq 0$), or vice versa; *provided that* processes p' and p'' satisfy the conservation laws of energy, momentum, angular momentum, and all 'charges'. Moreover, the Superluminal boost will transform B's into T's (i.e., will change $s, t, \ldots$ into $-s, -t, \ldots$). As before, scatterings p'' are the $\hat{C}\hat{P}\hat{T}$-ed ones of scatterings p'.

One could analogously consider the effect of GLT's on 'mixed' interactions between both T's and B's (or T's having $|V_x| < c$). Those interactions are interesting *also* for the problem of tachyons and 'virtual particles' (see e.g., Recami, 1969a).

4.3. *Case of Reactions Among Tachyons (With $|V_x| > c$)*

Let us confine ourselves, for simplicity, to interaction processes when the sum of initial and final particle number is *four*. From point (1) of the previous sub-section, it follows—*for an interaction between tachyons* (with $|V_x| > c$)—that (Arons & Sudarshan, 1968; Dhar & Sudarshan, 1968; Baldo *et al.*, 1970): The same scattering amplitude, governing process

$$A(\mathbf{p}_A, q_A, \lambda_A) + B(\mathbf{p}_B, q_B, \lambda_B) \to C(\mathbf{p}_C, q_C, \lambda_C) + D(\mathbf{p}_D, q_D, \lambda_D) \tag{4.1.2}$$

or process:

$$\bar{C}(\mathbf{p}_C, -q_C, -\lambda_C) + \bar{D}(\mathbf{p}_D, -q_D, -\lambda_D) \to \bar{A}(\mathbf{p}_A, -q_A, -\lambda_A) + \bar{B}(\mathbf{p}_{\bar{B}}, -q_B, -\lambda_B) \tag{4.1.2$'$}$$

where $\mathbf{p}, q, \lambda$ are trimomentum, 'charge' and helicity respectively, is required by extended Relativity to govern also processes (as well as their totally $\hat{C}\hat{P}\hat{T}$-ed versions) like:

$$A + \bar{C}(-\mathbf{p}_{\bar{C}}, -q_C, +\lambda_C) \to \bar{B}(-\mathbf{p}_{\bar{B}}, -q_B, +\lambda_B) + D \tag{4.3.1}$$

(and similar particle permutations); and also processes like

$$A \rightarrow \bar{B}(-\mathbf{p}_{\bar{B}}, q_B, +\lambda_B) + C + D \tag{4.3.2}$$

$$A + B + \bar{C}(-\mathbf{p}_{\bar{C}}, -q_C, +\lambda_C) \rightarrow D \tag{4.3.3}$$

(and similar particle permutations); provided that they are kinematically allowed. Notice that, e.g., in equation (4.3.1), the trimomenta of A and D are the transformed ones of $\mathbf{p}_A$, $\mathbf{p}_D$ by means of the subluminal boost L under consideration. Besides, *trimomenta appearing in* $\bar{C}(-\mathbf{p}_{\bar{C}}, \ldots)$, $\bar{B}(-\mathbf{p}_{\bar{B}}, \ldots)$, *as well as in the following, record only the versus of the transformed trimomenta with respect to the original ones* $\mathbf{p}_C$, $\mathbf{p}_B$, rather than assigning them a precise value.

Moreover, it is easy to find out relations between a reaction p among B's and reactions among the *corresponding* T's (i.e., reactions obtained by applying a SLT to reaction p). For instance, let us choose the first observer in the c.m.s. of reaction (4.1.2) among four B's. Previous point (2) tells us that: If $A = A(s, t, \ldots)$ is the scattering amplitude of

$$A(\mathbf{p}_A, q_A, \lambda_A) + B(\mathbf{p}_B, q_B, \lambda_B) \rightarrow C(\mathbf{p}_C, q_C, \lambda_C) + D(\mathbf{p}_D, q_D, \lambda_D) \tag{4.1.2}$$

then *the same* function $A = A(-s, -t, \ldots)$ will be the scattering amplitude of the following processes and of their similar particle-permutations as well as of all their $\hat{C}\hat{P}\hat{T}$-ed versions, now considered as reactions among T's, provided that they are kinematically allowed:

$$A + \bar{C}(\mathbf{p}_{\bar{C}}, -q_C, -\lambda_C) \rightarrow \bar{B}(\mathbf{p}_{\bar{B}}, -q_B, -\lambda_B) + D; \tag{4.3.4}$$

$$\begin{aligned} &\bar{C}(-\mathbf{p}_{\bar{C}}, -q_C, +\lambda_C) + \bar{D}(\mathbf{p}_{\bar{D}}, -q_D, -\lambda_D) \\ &\rightarrow \bar{A}(\mathbf{p}_{\bar{A}}, -q_A, -\lambda_A) + \bar{B}(-\mathbf{p}_{\bar{B}}, -q_B, +\lambda_B); \end{aligned} \tag{4.3.5}$$

$$\bar{C}(+\mathbf{p}_{\bar{C}}, -q_C, -\lambda_C) \rightarrow \bar{A}(-\mathbf{p}_{\bar{A}}, -q_A, +\lambda_A) + \bar{B}(\mathbf{p}_{\bar{B}}, -q_B, -\lambda_B) + D; \tag{4.3.6}$$

$$A + \bar{C}(\mathbf{p}_{\bar{C}}, -q_C, -\lambda_C) + \bar{D}(-\mathbf{p}_{\bar{D}}, -q_D, +\lambda_D) \rightarrow \bar{B}(\mathbf{p}_{\bar{B}}, -q_B, -\lambda_B) \tag{4.3.7}$$

where—as before—the 'trimomenta' appearing inside the brackets record only the *versus* of the transformed (*tachyonic*) trimomenta with respect to the original (bradyonic) ones.

4.4. *Case of Interaction Among Bradyons. Conclusions*

It is noticeable that, by using SLT's, we may get results *holding for bradyons* (Atkinson, 1973). In fact, if two processes among B's (e.g., an interaction and the *crossed* one (Chew, 1962; Hagedorn, 1963; Roman, 1969)) are different reactions p_1, p_2, as seen by us, but they are seen as the *same* interaction $d_S \equiv d_1 \equiv d_2$ (among T's) by two different Superluminal observers, S_1, S_2 (cf. point (2) of Section 4.2), *then* we may conclude the following. We may get the scattering amplitude of p_1, i.e., $A(p_1)$, by applying the SLT$(S_1 \to s_0) \equiv L_1$ to the amplitude $A_1(d_1)$ found by S_1 when observing scattering p_1:

$$A(p_1) = L_1[A_1(d_1)]$$

Conversely, we may get the scattering amplitude of p_2, i.e., $A(p_2)$, by applying the SLT$(S_2 \to s_0) \equiv L_2$ to the amplitude $A_2(d_2)$ found by S_2 when observing scattering p_2:

$$A(p_2) = L_2[A_2(d_2)]$$

But, since by hypothesis

$$A_1(d_1) = A_2(d_2) = A(d_S) \tag{4.4.1}$$

it follows that

$$A(p_1) = A(p_2) \tag{4.4.2}$$

for all reactions among B's satisfying the initial hypothesis.

From point (2) of Section 4.2 and from the previous sub-section (equations (4.1.2), (4.3.4)-(4.3.7)), it follows that *extended Relativity requires the scattering amplitude $A(s, t, \ldots)$ to be given by the same function of the kinematical variables for the following reactions* (We limit ouselves, as before, for simplicity, to four-body processes):

(i) the process

$$A(\mathbf{p}_A, q_A, \lambda_A) + B(\mathbf{p}_B, q_B, \lambda_B) \to C(\mathbf{p}_C, q_C, \lambda_C) + D(\mathbf{p}_D, q_D, \lambda_D) \tag{4.1.2}$$

(ii) *and* the totally $\hat{C}\hat{P}\hat{T}$-ed one (see equation (4.1.2′)):

(iii) *and* the *crossed processes* like equation (4.3.4);

(iv) *and* the partly $\hat{C}\hat{P}\hat{T}$-ed and partly $\hat{C}\hat{T}$-ed processes like equation (4.3.5);

(v) the 'decay processes' of the type of equation (4.3.6), when allowed;

(vi) the 'formation processes' of the type of equation (4.3.7), when allowed.

Of course, the kinematical variables $s, t \ldots$ will have for the different processes the different meanings and values pertaining to them for the new processes.

We conclude that:

(1) We have derived *crossing relations* (Chew, 1962; Hagedorn, 1963; Roman, 1969), even for B's, from mere extended Relativity;

(2) New *'crossing-type'* relations are required by PR: such relations may well serve as tests for relativistic covariance of 'force fields' like 'strong interactions' and particularly 'weak interactions' or possible, new 'interaction fields' (which *a priori* are not relativistically covariant);

(3) Extended Relativity itself requires that the same function $A(s, t, \ldots)$ gives the scattering amplitudes of different processes (as channels s, t, u, ... of a four-bradyon reaction) in correspondence to their physical domains of s, t, ...:

Therefore, in this framework, *'analyticity'* (Chew, 1962; Hagedorn, 1963; Roman, 1969) *is unnecessary, and better substituted by the G-covariance requirement.*

To further clarify the physical meaning of our procedure, let us lastly observe the following. In a two-body to two-body scattering between elementary particles (let us consider it as the reaction s-channel), the square t of the transferred four-momentum is well known (Berestetsky *et al.*, 1971) to be generally *negative* (Chew, 1962; Hagedorn, 1963; Ferretti & Verde, 1966; Hadjioannou, 1966; Roman, 1969; Verde):†

$$t \equiv p^2 < 0 \tag{4.4.3}$$

This quantity is, moreover, known (Berestetsky *et al.*, 1971) *both* to become positive *and* to change its meaning (e.g., from 'square momentum transfer' to 'square total energy') when passing from the s-channel to the t-channel.‡

This accords to the above-seen fact that a SLT may transform a reaction (*among bradyons*) into the *crossed* one (*among tachyons*).

These points‡ help clarify why recourse to *two* Superluminal Lorentz Transformations is needed for demonstrating *crossing relations* among bradyons.

Acknowledgments

The authors are grateful to Prof. A. Agodi for very fruitful comments. Thanks are also due to Profs. G. Arcidiacono, M. Baldo, R. de Benedetti for useful discussions, and to Dr. R. L. Baldini, Mrs. G. Giuffrida, Mr. S. Leotta, Dr. S. Rodono and Mr. P. Zingale for their kind collaboration.

References

Agodi, A. (1973). *Lectures in Theoretical Physics,* Catania, unpublished.
Antippa, A. F. and Everett, A. E. (1973). *Physical Review*, **8**, 2352.
Arons, M. E. and Sudarshan, E. C. G. (1968). *Physical Review*, **173**, 1622.
Atkinson, D. A. (1973). Preprint, Tekn. Technology Univ.
Baldo, M. and Recami, E. (1969). *Letters Nuovo Cimento*, **2**, 643.
Baldo, M., Fonte, G. and Recami, E. (1970). *Letters Nuovo Cimento,* **4**, 241.

† Since 1968, within the framework of 'peripheral models' (e.g., the one-particle-exchange model), the possibility that usual 'virtual particles' be actually considered as tachyons has been suggested in Recami (1969a, 1969b, 1970, and in Olkhovsky & Recami (1969)). See also Sudarshan (1969a, 1969b, 1972), Baldo *et al.* (1970), Gleeson *et al.* (1972) and van der Spuy (1973).

‡ Since GLT's may also change the observed *channel*, in order to avoid confusion it is better to consider the action of GLT's on *physical quantities* as the 'observed total entering four-momentum' or the 'observed transferred four-momentum', rather than on their *formal* expressions according to a certain observer (i.e., relative to a certain channel). In fact, such expressions for the above-mentioned *physical* quantities may well change when changing observer (i.e., when the observed channel changes).

Berestetsky, V., Lifshitz, E. M. and Pitaevsky, P. (1971). *Relativistic Quantum Theory*, p. 34, London.
Bilaniuk, O. M. P., Deshpande, V. K. and Sudarshan, E. C. G. (1962). *American Journal of Physics*, **30**, 718.
Chew, G. (1962). *S-Matrix Theory of Strong Interactions*, Benjamin Pub., N.Y.
Csonka, P. L. (1969). *Physical Reviews*, **180**, 1266.
Dhar, T. and Sudarshan, E. C. G. (1968). *Physical Review*, **174**, 1808.
Dirac, P. A. M. (1930). *Proceedings of the Royal Society (London)*, **A126**, 360.
Fabri, E. (1959). *Nuovo Cimento*, **14**, 1130.
Feinberg, G. (1967). *Physical Review*, **159**, 1089.
Ferretti, I. and Verde, M. (1966). *Atti dell'Accademia delle scienze. Torino*, p. 318. Verde, M. (unpublished).
Feynamn, R. P. (1949). *Physical Review*, **76**, 749, 769.
Gleeson, A. M., Gundzik, M. G., Sudarshan, E. C. G. and Pagnamenta, A. (1972). *Physical Review*, **D6**, 807.
Glück, M. (1969). *Nuovo Cimento*, **A62**, 791.
Goldoni, R. (1973). *Nuovo Cimento*, **A14**, 501.
Hadjioannou, F. T. (1966). *Nuovo Cimento*, **44**, 185.
Hagedorn, R. (1963). *Relativistic Kinematics*, Benjamin Pub.
Lee, T. D. and Yang, C. N. (1957). *Physical Review*, **105**, 1671.
Messiah, A. (1965). *Quantum Mechanics*, Volume I, Chapter 10. North-Holland Pub., Amsterdam.
Mignani, R. and Recami, E. (1973a). 'Classical Theory of Tachyons (Special Relativity Extended to Superluminal Frames and Objects)', preprint IFTUC-PP/387, Catania, to appear in *Rivista Nuovo Cimento*, and references therein.
Mignani, R. and Recami, E. (1973b). *Nuovo Cimento*, **A14**, 169.
Mignani, R. and Recami, E. (1973c). *Nuovo Cimento*, **A16**, 208.
Mignani, R. and Recami, E. (1973d). *Letters Nuovo Cimento*, **7**, 388; **8**, 780; **9**, 362, 367, 479; **11**, 417; *Nuovo Cimento*, **B21**, 210.
Olkhovsky, V. S. and Recami, E. (1969). *Nuovo Cimento*, **A63**, 814.
Olkhovsky, V. S. and Recami, E. (1971). *Letters Nuovo Cimento*, **1**, 165.
Parker, L. (1969). *Physical Review*, **188**, 2287.
Pauli, W. (1921). *Relativitätstheorie*, Leipzig.
Recami, E. (1969a). *Giornale di Fisica*, **10**, 195.
Recami, E. (1970). *Atti dell'Accademia nazionale dei Lincei. Rendiconti*, **49**, 77.
Recami, E. (1973). I Tachioni, in *Enciclopedia EST-Mondadori, Annuario 73*, p. 85. Milano.
Recami, E. and Mignani, R. (1972). *Letters Nuovo Cimento*, **4**, 144.
Recami, E. and Mignani, R. (1973). *Letters Nuovo Cimento*, **8**, 110.
Recami, E. and Mignani, R. (1974). *Rivista Nuovo Cimento*, **4**, 209, 398, and references therein.
Roman, P. (1969). *Introduction to Quantum Field Theory*, J. Wiley Inc., New York.
Sakurai, J. J. (1964). *Invariance Principles and Elementary Particles*, p. 137. Princeton.
Sakurai, J. J. (1964). *Invariance Principles and Elementary Particles*, Chapter 7. Princeton.
Stückelberg, E. C. G. (1941). *Helvetica Physica Acta*, **14**, 32L, 588.
Sudarshan, E. C. G. (1968). *Proceedings of the Indian Academy of Sciences*, **67**(5A), 284.
Sudarshan, E. C. G. (1969a). *Arkiv für Physik*, **39**(40), 585.
Sudarshan, E. C. G. (1969b). *Proceedings of the Indian Academy of Sciences*, **69(3A)**, 133; and in *1968 Proceedings of the VIII Nobel Symposium (Elementary Particle Theory)*, Swartholm, N. ed., p. 335. J. Wiley Pub., New York and Almqvist and Wiksell Pub., Stockholm.
Sudarshan, E. C. G. (1970). in *Symposia in Theor. Phys. and Math.* Vol. 10, p. 129, Plenum Press.
Sudarshan, E. C. G. (1972). Preprint CPT-166, Austin; (1968). Preprint NYO-3399-191/SU-1206-191, Syracuse.